

6.5 p1

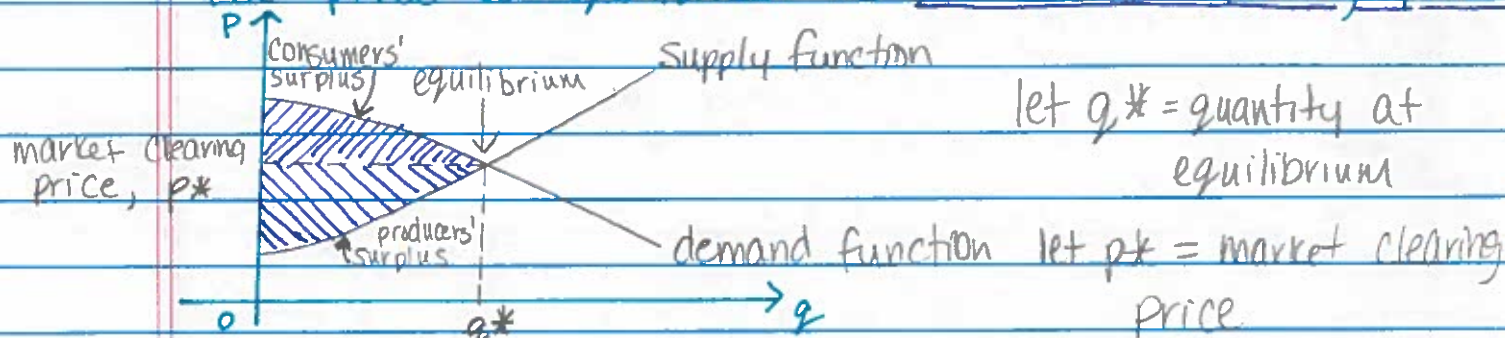
MA121

June 19/18

Consumer and Producer Surplus

Recall that the intersection between the demand function and the supply function is the equilibrium.

The price at equilibrium is the market clearing price.



Consumer Surplus

$$CS = \int_0^{q^*} (\text{demand function} - p^*) dq$$

Producer Surplus

$$PS = \int_0^{q^*} (p^* - \text{supply function}) dq$$

ex 4p 204

$$\text{Supply} = p = 0.1q + 3 \quad \text{demand} = p = \frac{10q + 190}{q + 10}, \quad 0 \leq q \leq 200$$

i) find equilibrium point. Equilibrium occurs when

$$\text{Supply} = \text{demand}$$

$$\Rightarrow 0.1q + 3 = \frac{10q + 190}{q + 10} \Rightarrow (0.1q + 3)(q + 10) = 10q + 190$$

$$\Rightarrow 0.1q^2 + 4q + 30 - 10q - 190 = 0 \Rightarrow 0.1q^2 - 6q - 160 = 0$$

$$\Rightarrow q^2 - 60q - 1600 = 0 \Rightarrow (q - 80)(q + 20) = 0$$

$$\Rightarrow q = 80, \quad q = -20. \text{ accept } q = 80, \text{ reject } q = -20.$$

$\therefore q^* = 80$ as the equilibrium quantity.

ii) To find the market clearing price, use either the Supply function...

$$\begin{aligned} p^* &= (0.1q^* + 3) \\ &= (0.1)(80) + 3 \\ &= \$11.00 \end{aligned}$$

6.5 p2

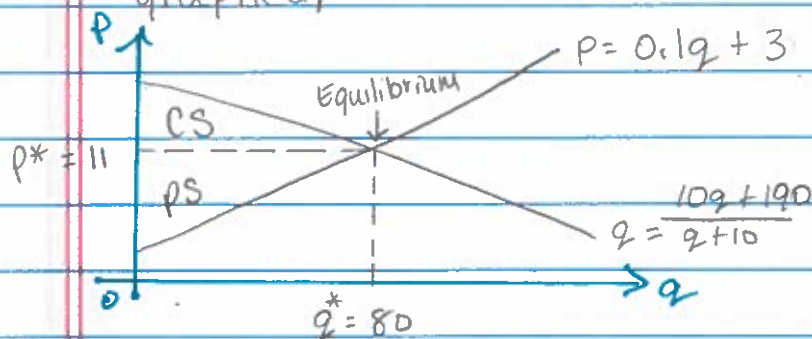
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June 19/8

or the demand function..

$$p^* = \frac{10q^* + 190}{q^* + 10} = \frac{10(80) + 190}{80 + 10} = \frac{990}{90} = \$11.00.$$

graphed,



Consumers' Surplus:

$$CS = \int_0^{Q^*} (\text{demand function} - p^*) dq$$

$$\Rightarrow CS = \int_0^{80} \left(\frac{10q + 190}{q + 10} - 11 \right) dq$$

$$\Rightarrow CS = \int_0^{80} \left(\frac{10(q+10) - 100 + 190}{q + 10} - 11 \right) dq$$

$$\Rightarrow \int_0^{80} \left(\frac{10(q+10) + 90}{q + 10} - 11 \right) dq$$

$$\Rightarrow \int_0^{80} \left(10 + \frac{90}{q + 10} - 11 \right) dq$$

$$= \int_0^{80} \left(\frac{90}{q + 10} - 1 \right) dq$$

-10	10	190
0	0	-100
10	10	90

$$= \left[90 \ln |q + 10| - q \right]_0^{80}$$

$$= \left[90 \ln |80 + 10| - 80 \right]_0^{80} - \left[90 \ln |0 + 10| - 0 \right]$$

$$= 90 \ln 90 - 80 - 90 \ln 10 = 90 (\ln 90 - \ln 10) - 80$$

$$= 90 \ln \frac{90}{10} - 80 = 90 \ln 9 - 80$$

$$\approx \$117.75$$

6.5 p3

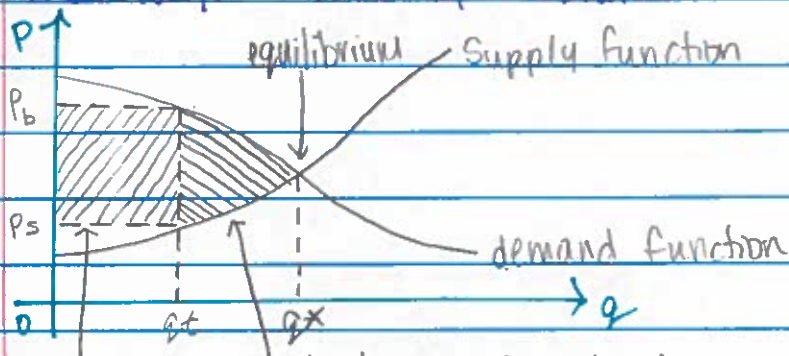
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June 1918

Producers' Surplus

$$\begin{aligned}
 PS &= \int_0^{q^*} (p^* - \text{Supply function}) dq \\
 &= \int_0^{80} (11 - (0.1q + 3)) dq = \int_0^{80} (11 - 0.1q - 3) dq \\
 &= \int_0^{80} (8 - 0.1q) dq \\
 &= [8q - 0.05q^2]_0^{80} \\
 &= [8(80) - 0.05(80)^2]_0^{80} - [8(0) - 0.05(0)^2] \\
 &= 640 - 320 - 0 \\
 &= \$320.00
 \end{aligned}$$

Deadweight loss of Taxation



$$\begin{aligned}
 \text{tax revenue} &= q^*(P_b - P_s) \\
 \text{deadweight loss due to taxation (DLT)} &= \int_0^{q^*} (\text{demand funct.} - \text{supply funct.}) dq
 \end{aligned}$$

14 p 204

$$\text{Supply} = P = 5 + 0.025q, \text{ demand} = P = \frac{6q + 400}{q + 20}, \quad 0 \leq q \leq 500$$

Equilibrium occurs when supply = demand:

$$\Rightarrow 5 + 0.025q = \frac{6q + 400}{q + 20} \Rightarrow (5 + 0.025q)(q + 20) = 6q + 400$$

$$\Rightarrow 5q + 100 + 0.025q^2 + 0.5q = 6q + 400$$

$$\Rightarrow 0.025q^2 + 5.5q + 100 - 6q - 400 = 0$$

$$\Rightarrow 0.025q^2 - 0.5q - 300 = 0$$

$$\Rightarrow \frac{0.025q^2 - 0.5q - 300}{0.025} = 0$$

$$\Rightarrow q^2 - 20q - 12000 = 0$$

$$\Rightarrow (q - 120)(q + 100) = 0$$

$$\Rightarrow q = 120, \quad q = -100 \quad \text{reject } q = -100, \text{ accept } q = 120.$$

6.5 p4

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∴ $q^* = 120$, and the market clearing price is

$$p^* = 5 + 0.025q^* \text{ (supply function)}$$

$$\Rightarrow p^* = 5 + 0.025(120) = \$8.00$$

or, with the demand function,

$$p^* = \frac{6q^* + 400}{q^* + 20}$$

$$\Rightarrow p^* = \frac{6(120) + 400}{120 + 20} = \frac{720 + 400}{140} = \frac{1120}{140} = \$8.00$$

We are told that the government imposes a tax and the quantity sold after taxes becomes $q^t = q^* - 10$

$$\Rightarrow q^t = q^* - 10 \Rightarrow q^t = 120 - 10 \Rightarrow q^t = 110.$$

So the price that buyers pay after taxes

$$P_b = \text{price at } q^t = \frac{6(110) + 400}{110 + 20} = \$8.15$$

and the price that suppliers get after taxes

$$\Rightarrow P_s = \text{price at } q^t = 5 + 0.025(110) = \$7.75$$

$$\text{Tax revenue} = q^t (P_b - P_s)$$

$$\Rightarrow (110)(8.15 - 7.75)$$

$$\Rightarrow (110)(0.40) = \$44.00$$

Deadweight loss due to taxation...

$$DLT = \int_{q^t}^{q^*} (\text{dem. fun.} - \text{sup. fun.}) dq$$

$$= \int_{110}^{120} \left(\frac{6q + 400}{q + 20} - (5 + 0.025q) \right) dq$$

$$= \int_{110}^{120} \left(6 + \frac{280}{q + 20} - 5 - 0.025q \right) dq$$

$$= \int_{110}^{120} \left(1 + \frac{280}{q + 20} - 0.025q \right) dq$$

$$= \left[q + 280 \ln |q + 20| - 0.0125 \left(\frac{q^2}{2} \right) \right]_{110}^{120}$$

$$= \left[q + 280 \ln |q + 20| - 0.0125 q^2 \right]_{110}^{120}$$

$$= [120 + 280 \ln(120 + 20) - 0.0125(120)^2] - [110 + 280 \ln(110 + 20) - 0.0125(110)^2]$$

$$\approx \$2.00$$

Q.5 p5

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June 18/19

mean value of $y = f(x)$ on $a \leq x \leq b$

$$MV = \frac{1}{b-a} \int_a^b f(x) dx$$

ex 13 p 204

$$f(x) = \sqrt{x+1}, \quad 0 \leq x \leq 4$$

$$MV = \frac{1}{4-1} \int_1^4 (\sqrt{x} + 1) dx = \frac{1}{3} \int_1^4 (x^{1/2} + 1) dx$$

$$= \frac{1}{3} \left[\frac{x^{3/2}}{3/2} + x \right]_1^4 = \frac{1}{3} \left[\frac{2}{3} \sqrt{x^3} + x \right]_1^4$$

$$= \frac{1}{3} \left[\left(\frac{2}{3} \sqrt{4^3} + 4 \right) - \left(\frac{2}{3} \sqrt{1^3} + 1 \right) \right]$$

$$= \frac{1}{3} \left[\frac{2}{3} (8) + 4 - \frac{2}{3} - 1 \right] = \frac{1}{3} \left[\frac{16}{3} + 4 - \frac{2}{3} - 1 \right]$$

$$= \frac{1}{3} \left[\frac{14}{3} + 3 \right] = \frac{1}{3} \left(\frac{23}{3} \right) = \frac{23}{9}$$

Antiderivatives from page 197...

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$